S 169 1) b) $a_{n+1} = a_n^2 + 1/4$; $a_1 = 0$; $n \geq 1$

$n = 1$ $\quad a_{1+1} = a_1^2 + 1/4 \quad a_2 = a_1^2 + 1/4 = 1/4$

$n = 2$ $\quad a_{2+1} = a_2^2 + 1/4 \quad a_3 = a_2^2 + 1/4 = 5/16$

Behauptung : $a_{n+1} > a_n$

$n = 1$: $\quad a_{1+1} > a_1 \quad a_2 > a_1 \quad 1/4 > 0$ ✓

$n+1$: $\quad a_{n+2} > a_{n+1}$

$$a_{n+1}^2 + 1/4 > a_n^2 + 1/4 \quad | - 1/4$$

$$a_{n+1}^2 > a_n^2 \quad | \sqrt{\ }$$

$$a_{n+1} > a_n$$

a_{n+1} ist streng monoton steigend, wodurch $a_1 = 0$ untere Schranke sein muss.

$a_{n+1} = a_n^2 + 1/4$ ✓

$a_n < 1/2$ → Behauptung

$n = 1$ $\quad a_1 < 1/2 \quad 0 < 1/2$ ✓

n+1:

$a_n < 1/2 \quad | \uparrow^2$

$a_n^2 < 1/4 \quad | + 1/4$

$\underbrace{a_n^2 + 1/4} < 1/4 + 1/4 = 1/2$

$a_{n+1} < 1/2$

optimale Schranke

$a_n^2 + 1/4 = a_{n+1}$

Behauptung: $a_n < 10 \quad |()^2$

$$a_n^2 < 100 \quad |+1/4$$

$$a_n^2 + 1/4 < 100\tfrac{1}{4}$$

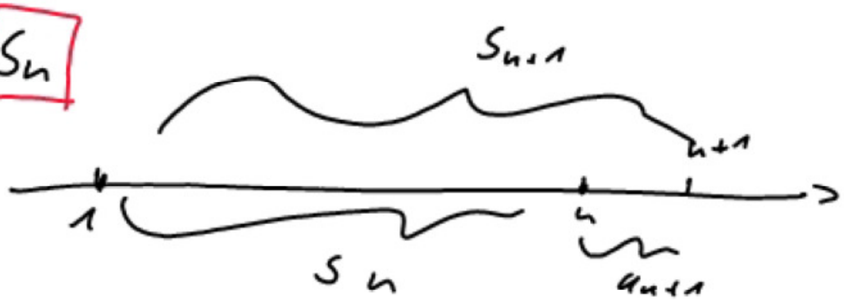
$$a_{n+1} < 100\tfrac{1}{4} \quad , \text{da} \quad a_n < 10$$

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n \quad \wedge \quad \lim_{n\to\infty} a_n = r$$

$$r^2 + 1/4 = r \quad |-r$$

$$r^2 - r + 1/4 = (r - 1/2)^2 = 0 \qquad r = 1/2$$

$$\sum_{k=1}^{n} a_k = \boxed{S_n}$$



<u>$n+1$</u>

$$\sum_{k=1}^{n+1} a_k = \underbrace{a_1 + a_2 + a_3 + \dots + a_n}_{} + a_{n+1} = S_{n+1}$$

$$\boxed{S_n + a_{n+1} = S_{n+1}}$$

S 17 3 b) $\sum_{\underbrace{k=1}_{\text{Startwert}}}^{n} \Big(\underbrace{\tfrac{k}{2^k}}_{\text{Folge } a_k}\Big) = \underbrace{2 - \frac{n+2}{2^n}}_{\text{Summenformel } S_n}$

$n = 1$: $\underbrace{\sum_{k=1}^{1} k/2^k}_{a_1 = 1/2} = S_1$

$= 2 - \frac{1+2}{2} = 2 - 3/2 = 1/2$ ✓

Prämisse: Es gilt $\sum_{k=1}^{n} (k/2^k) = 2 - \frac{n+2}{2^n}$

$n+1$: $\sum_{k=1}^{n+1} (k/2^k) = \underbrace{\sum_{k=1}^{n} (k/2^k)}_{S_n} + a_{n+1} = S_{n+1}$

$$\underbrace{2-\frac{n+2}{2^n}}_{S_n}+\underbrace{\frac{n+1}{2^{n+1}}}_{a_{n+1}}=\underbrace{2-\frac{(n+1)+2}{2^{n+1}}}_{S_{n+1}} \quad \Big| -2$$

$$-\frac{n+2}{2^n}+\frac{n+1}{2^{n+1}}=-\frac{n+3}{2^{n+1}} \quad \Big| \cdot 2^{n+1}=2\cdot 2^n$$

$$-2(n+2)+n+1=-(n+3)$$

$$\left.\begin{array}{l}-2n-4+n+1\\ -n-3\end{array}\right\}=-n-3 \quad \Big| +n+3$$

$$0=0 \quad \checkmark$$