a) $$\sum \frac{(3^k)^{2+3k}}{4\cdot(2k)!}$$

$$\lim_{k\to\infty} \frac{\left(3^{(k+1)}\right)^{2+3(k+1)}}{\cancel{4}\cdot(2\cdot(k+1))!} \cdot \frac{\cancel{4}\cdot(2k)!}{(3^k)^{2+3k}}$$

$$\lim_{k\to\infty} \frac{(2k)!}{(2k+2)!} \cdot \frac{3^{(k+1)\cdot(5+3k)}}{3^{k\,(2+3k)}}$$

$$\lim_{k\to\infty} \frac{(2k)!}{(2k+2)(2k+1)(2k)!} \cdot \frac{3^{3k^2+8k+5}}{3^{3k^2+2k}}$$

$$\lim_{k\to\infty} \frac{3^{6k+5}}{(2k+2)(2k+1)} = \left[\frac{\text{exp}}{\text{Quad}}\right] = \infty \Rightarrow \text{divergent}$$

5) $\sum -\frac{3k^3}{5k^{k+2}} = -\frac{3}{5} \cdot \sum \frac{k^3}{k^k k^2}$

$$\lim_{k\to\infty} \sqrt[k]{\frac{k^3}{k^k\, k^2}} = \lim_{k\to\infty} \frac{(\sqrt[k]{k})^3}{\sqrt[k]{k^k}\,(\sqrt[k]{k})^2} = \left[\frac{1^3}{k\cdot 1^2}\right] = 0 < 1$$

$$-\frac{3}{5} \cdot \sum \frac{k}{k^k}$$

$$\lim_{k\to\infty} \frac{(k+1)}{(k+1)^{k+1}} \frac{k^k}{k}$$

$$\lim_{k\to\infty} \frac{k^k}{(k+1)^k} \cdot \frac{1}{k+1} \cdot \frac{k+1}{k}$$

$$\lim_{k\to\infty} \left(\frac{k}{k+1}\right)^k \cdot \frac{1}{k} = \left[1 \cdot \frac{1}{\infty}\right] = 0 < 1 \checkmark$$

c) $\sum (-1)^{k+1} \cdot \frac{k^2-5}{2k^2-k^3}$

Leibniz $\Rightarrow \lim\limits_{k\to\infty} \frac{k^2-5}{2k^2-k^3}$

$$\lim_{k\to\infty} \frac{k^2\cdot(1-\overset{\to 0}{5/k^2})}{k^3\,(\underset{\downarrow\, 0}{2/k}-1)} = \left[\frac{1}{k\cdot(-1)}\right] = 0$$

d) $\sum\limits_{k=3}^{\infty} (1/2)^{k-2}$ $\nearrow 4\cdot\sum(1/2)^k$: konv. $\sqrt[k]{|1|^k}\cdot 1/2 < 1$ Nullfolge

$\xrightarrow{-3}$ $\sum\limits_{k=0}^{\infty} (1/2)^{(k+3)-2}$ $= 1/2 \cdot \sum\limits_{k=0}^{\infty} (1/2)^k = \frac{1}{2}\cdot\frac{1}{1-1/2} = 1$

e) $$\sum_{k=2}^{\infty} (-1)^k \frac{4 \cdot 5^{2k+2}}{(2k+1)!} \rightsquigarrow \sin(x)$$

$k=0$

$$\sum_{k=0}^{\infty} (-1)^k \frac{5^{2k+1}}{(2k+1)!} = \sin(5)$$

$$4 \cdot 5^1 \cdot \sum_{k=2}^{\infty} (-1)^k \cdot \frac{5^{2k+1}}{(2k+1)!}$$

$$20 \cdot \left[\sin(5) - \left[(-1)^0 \cdot \frac{5^{0+1}}{(0+1)!} + (-1)^1 \cdot \frac{5^{2+1}}{(2+1)!} \right] \right]$$

$$20 \cdot \left(\sin(5) - \left(5 - \frac{125}{6} \right) \right)$$