1) $\sum_{k=0}^{n} 5^k = \frac{5^{n+1}-1}{4}$

gg.: 1) $a_k = 5^k$

2) $S_n = \frac{5^{n+1}-1}{4}$

3) $k \geq 0 \Rightarrow n = 0$

$n = 0$: $a_0 = S_0$

$$5^0 = 1 = \frac{5^{0+1}-1}{4} = 4/4 = 1$$

$n+1$: $S_n + a_{n+1} = S_{n+1}$

$$\frac{5^{n+1}-1}{4} + 5^{n+1} = \frac{5^{(n+1)+1}-1}{4} \quad | \cdot 4$$

$$5^{n+1} - 1 + 4 \cdot 5^{n+1} = 5^{n+2} - 1 \quad | +1$$

$5^n = \beta$

$$5^{n+1} + 4 \cdot 5^{n+1} = 5 \cdot 5^{n+1} = 5^{n+2}$$

$$5 \cdot 5^n + 4 \cdot 5 \cdot 5^n = 5^2 \cdot 5^n$$

$$5 \cdot \beta + 20 \cdot \beta = 25 \cdot \beta$$

5) $\sum_{k=1}^{n} k^3 = \left(\frac{n\cdot(n+1)}{2}\right)^2$

geg: $a_k = k^3$

$S_n = \frac{n^2\cdot(n+1)^2}{4}$

$k \geq 1$

$n=1$ $\quad a_1 = S_1$

$1^3 = 1 = \frac{1\cdot 2^2}{4} = 4/4 = 1$ ✓

$n+1$: $\quad S_n + a_{n+1} = S_{n+1}$

$\frac{n^2\cdot(n+1)^2}{4} + 4(n+1)^3 = \frac{(n+1)^2\cdot(n+2)^2}{4} \quad |\cdot 4$ $\quad \beta = (n+1)$

$n^2\cdot\beta^2 + 4\beta^3 = \beta^2\cdot(n+2)^2$

$\beta^2\,(n^2+4\beta) = \beta^2\cdot(n+2)^2 \quad | : \beta^2 <> 0$

$n^2+4\cdot\beta = n^2+4\cdot(n+1) = (n+2)^2$

$n^2+4n+4 = n^2+4n+4$

$0 = 0$ ✓