$$\sum_{k=2}^{n} \overbrace{(k-1)\cdot\ln\left(\frac{k}{k-1}\right)}^{a_k} = \overbrace{n\cdot\ln(n) - \ln(n!)}^{S_n}$$

$k = 2$ $\qquad a_2 = S_2$

$$1\cdot\ln\left(\frac{2}{1}\right) = 2\cdot\ln(2) - \ln(2!)$$

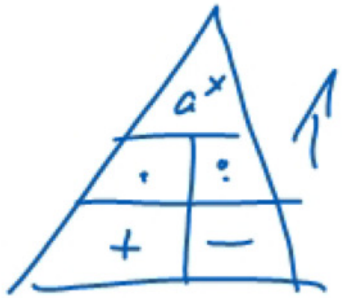
$$\ln(2) = \ln 2^2 - \ln 2 = \ln\frac{2^2}{2} = \ln 2$$

<u>$n+1$</u>

$$S_n + a_{n+1} = S_{n+1}$$

$$\overbrace{n\cdot\ln(n) - \ln(n!)} + n\cdot\ln\left(\frac{n+1}{n}\right) = (n+1)\cdot\ln(n+1) - \ln[(n+1)!]$$

$$\underline{n\cdot\ln(n)} - \ln(n!) + n\cdot\ln(n+1) \underline{- n\cdot\ln(n)} =$$



$$n\cdot\ln(n+1) + 1\cdot\ln(n+1) - \ln((n+1)\cdot n!)$$

$$-\ln(n!) + \underline{n\cdot\ln(n+1)} = \underline{n\cdot\ln(n+1)} + \ln(n+1) - [\ln(n+1) + \ln(n!)]$$

$$\underline{-\ln(n!)} = \underline{\ln(n+1) - \ln(n+1)} \underline{- \ln(n!)}$$

$$0 = 0 \quad ✓$$