I) a) $\sum_{k=1}^{\infty} \frac{(3^k)^{2+3k}}{4\cdot(2k)!}$ → Quotient : $\frac{a_{k+1}}{a_k}$

$$\lim_{k\to\infty} \frac{\left[3^{(k+1)}\right]^{2+3\cdot(k+1)}\ \cancel{4}\cdot(2k)!}{\cancel{4}\cdot[2\cdot(k+1)]!\ (3^k)^{2+3k}}$$

$$= \lim \frac{(2k)!}{(2k+2)!} \cdot \frac{3^{(k+1)\cdot(5+3k)}}{3^{k\cdot(2+3k)}}$$

$$\lim_{k\to\infty} \frac{(2k)!}{(2k+2)(2k+1)(2k)!} \cdot \frac{3^{3k^2+8k+5}}{3^{2k+3k^2}}$$

dominant

$$\lim_{k\to\infty} \left[\frac{1}{(2k+2)(2k+1)} \cdot 3^{6k+5}\right] = \left[\frac{\text{Potenz/EXP}}{\text{Quadrat.}}\right]$$

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \infty \nless 1$$ Divergenz

5) $$\sum_{k=1}^{\infty} -\frac{3\,k^3}{5\cdot k^{k+2}} = -\frac{3}{5}\cdot\sum \frac{k^3}{k^{k+2}}$$

Wurzel

$$\lim_{k\to\infty} \sqrt[k]{\frac{k^3}{k^k\cdot k^2}} = \lim_{k\to\infty} \frac{\overbrace{\sqrt[k]{k^3}}^{\to 1}}{\sqrt[k]{k^k}\;\underbrace{\sqrt[k]{k^2}}_{\to 1}}$$

$$\Rightarrow \lim_{k\to\infty}\left[\frac{1}{k}\right] = 0 < 1 \qquad ✓$$

Quotient

$$\lim_{k\to\infty} \frac{(k+1)^3 \cdot k^{k+2}}{(k+1)^{k+3}\cdot k^3}$$

$$\left(\frac{k+1}{k}\right)^3 \frac{k^2}{(k+1)^3}\;\frac{k^k}{(k+1)^k}$$

$$\downarrow$$

$$\underbrace{(1+1/k)^3}_{1^+}\cdot\underbrace{\frac{1}{k+1}}_{0}\cdot\underbrace{\left(\frac{k}{k+1}\right)^2}_{1^-}\cdot\underbrace{\left(\frac{k}{k+1}\right)^k}_{0} = 0 < 1$$

c) $\sum (-1)^{k+1} \left(\frac{k^2-5}{2k^2-k^3} \right)$

Leibniz : $\lim\limits_{k\to\infty} a_k = 0 = \left[\frac{k^2\cdot(\overbrace{1-\underbrace{5/k^2}_{\to 0}}^{1})}{\underbrace{k^3}_{k}(\underbrace{2/k}_{\to 0}-1)} \right]$

$\Rightarrow \left[\frac{1}{k\cdot(-1)} \right] \to 0^-$

d) $\sum\limits_{k=3}^{\infty} \left(1/2\right)^{k-2} = \sum\limits_{k=0}^{\infty} \left(1/2\right)^{(k+3)-2} = \sum\limits_{k=0}^{\infty} \left(1/2\right)^{k+1}$

$\sum\limits_{k=0}^{\infty} q^k = \frac{1}{1-q}$

$1/2 \cdot \frac{1}{1-1/2} \Rightarrow 1$