1) $$\sum_{k=2}^{5} \left(\tfrac{1}{4}\right)^k = \sum_{k=0}^{5} \left(\tfrac{1}{4}\right)^k - \left[\left(\tfrac{1}{4}\right)^0 + \left(\tfrac{1}{4}\right)^1 \right]$$

$$\frac{1-\left(\tfrac{1}{4}\right)^6}{1-\tfrac{1}{4}} - \left(1 + \tfrac{1}{4}\right) = \tfrac{4}{3} \cdot \left(1 - \left(\tfrac{1}{4}\right)^6\right) - \tfrac{5}{4}$$

2) $$\sum_{k=3}^{6} \left(\tfrac{2}{3}\right)^{k-1} = \tfrac{3}{2} \cdot \sum_{k=3}^{6} \left(\tfrac{2}{3}\right)^k$$

$$= \tfrac{3}{2} \cdot \left(\sum_{k=0}^{6} \left(\tfrac{2}{3}\right)^k - \sum_{k=0}^{2} \left(\tfrac{2}{3}\right)^k \right)$$

$$= \tfrac{3}{2} \cdot \left[\frac{1-\left(\tfrac{2}{3}\right)^7}{1-\tfrac{2}{3}} - \frac{1-\left(\tfrac{2}{3}\right)^3}{1-\tfrac{2}{3}} \right]$$

$$= \tfrac{9}{2} \cdot \left(1 - \left(\tfrac{2}{3}\right)^7 - 1 + \left(\tfrac{2}{3}\right)^3\right)$$

$$= \tfrac{9}{2} \cdot \left(\left(\tfrac{2}{3}\right)^3 - \left(\tfrac{2}{3}\right)^7\right)$$

$$\left(\tfrac{2}{3}\right)^k \cdot \left(\tfrac{2}{3}\right)^{-1}$$

$$= \tfrac{3}{2}$$

3) $\sum_{k=2}^{6} 1/4 \cdot 0{,}1^{k-2} = 1/4 \cdot 100 \cdot \sum_{k=2}^{6} (1/10)^{k}$

$25 \cdot \sum_{k=0}^{4} (1/10)^{k+2} = 25 \cdot 1/100 \cdot \frac{1-(1/10)^{5}}{1-1/10}$

$= 1/4 \cdot 10/9 \cdot (1-(1/10)^{5})$

8) b) $1024 \cdot \sum_{k=1}^{4} (1/2)^{2\cdot(2+k)} = 2^{10} \sum_{k=1}^{4} (1/2)^{4} \cdot (1/2)^{2k}$

$2^{10} \cdot \left(\frac{1}{2}\right)^{4} \cdot \left[\sum_{k=0}^{4} (1/4)^{k} - 1\right]$ $\quad \frac{2^{8}}{3} \cdot \frac{255}{2^{10}} = \frac{85}{4}$

$2^{6} \cdot \left(\frac{1-(1/4)^{5}}{1-1/4} - 1\right) = 2^{6} \cdot \left[\frac{1-1/1024}{3/4} - 1\right]$

$2^{6} \cdot 4/3 \cdot (1 - 1/1024 - 3/4) = \frac{2^{8}}{3} \cdot \left(\frac{1024-1-768}{1024}\right)$

$$\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$$

a) $$\sum_{k=3}^{\infty} \frac{3^k}{k!} \to e^3$$

$$e^3 - \left[\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}\right] = e^3 - 8{,}5$$

b) $$\sum_{k=2}^{5} \left(\tfrac{2}{3}\right)^k = \left(\tfrac{2}{3}\right)^2 + \left(\tfrac{2}{3}\right)^3 + \left(\tfrac{2}{3}\right)^4 + \left(\tfrac{2}{3}\right)^5$$

$$= \sum_{k=0}^{3} \left(\tfrac{2}{3}\right)^{k+2} = \frac{4}{9} \cdot \frac{1-\left(\tfrac{2}{3}\right)^4}{1-\tfrac{2}{3}}$$

$$\frac{4}{9} \cdot 3 \cdot \left(1 - \frac{16}{81}\right) = \frac{4}{3} \cdot \frac{65}{81}$$

c) $$\sum_{k=1}^{\infty} \frac{4}{k^2} - \sum_{k=0}^{3} 5^k = 4 \cdot \frac{\pi^2}{6} - (1 + 5 + 25 + 125)$$

$$= \tfrac{2}{3}\pi^2 - 156$$

d) $4 \cdot \sum_{k=3}^{8} 4 \cdot (1/2)^{2k} = 16 \cdot \sum_{k=0}^{5} (1/2)^{2 \cdot (k+3)}$

$= 16 \cdot \sum_{k=0}^{5} (1/4)^{k} \cdot (1/2)^{6}$

$= \frac{1}{4} \cdot \frac{1-(1/4)^{6}}{1-1/4} = 1/3 \cdot (1-(1/4)^{6})$

e) $\sum_{k=4}^{\infty} (1/3)^{2k+1} = 1/3 \cdot \sum_{k=4}^{\infty} (1/9)^{k} = \frac{1}{3} \cdot \sum_{k=0}^{\infty} (1/9)^{k+4}$

$1/3 \cdot (1/9)^{4} \cdot \frac{1}{1-1/9} = \frac{1}{3} \cdot \frac{1}{9^{4}} \cdot \frac{9}{8} = \frac{1}{8 \cdot 3^{7}}$

f) $\sum_{k=2}^{\infty} (-1)^{k} \cdot \frac{3 \cdot x^{2k}}{(2k)!} = 3 \cdot \left[\cos(x) - \left((-1)^{1} \cdot \frac{x^{2}}{2!}\right)\right]$

$= 3 \cdot (\cos(x) + 1/2\, x^{2})$

a) $\sum_{k=1}^{\infty} 3/k^2+1 \Rightarrow \frac{1}{k^2+1} < \frac{1}{k^2} \rightarrow$ Konvergent VGL.

$\sum \frac{2}{k!} = 2 \cdot \sum \frac{1^k}{k!} \approx 2 \cdot e$

b) $\sum (-1)^{k+1} \cdot \frac{1}{\sqrt{4k+1}}$

↳ Alternierungsreihe => Leibniz ~ ⊕ Konvergenz

$\lim_{k \to \infty} \frac{1}{\sqrt{4k+1}} = \left[\frac{1}{\sqrt{\infty}}\right] = \left[\frac{1}{\infty}\right] = 0 \Rightarrow$ Nullfolge

c) $\sum \frac{5^{-k+1} \cdot k^2}{3k-2}$

$$\lim_{k \to \infty} \sqrt[k]{\frac{5^{-k} \cdot 5 \cdot k^2}{3k-2}} = \frac{\sqrt[k]{5^{-k}} \cdot \sqrt[k]{5k^2}}{\sqrt[k]{3k-2}}$$

$$(5 \cdot k^2)^{1/k} = (5k^2)^0 = 1$$

$$\Rightarrow \frac{5^{-1} \cdot 1}{1} = 1/5 < 1 \quad ✓$$

$$\sum \frac{5 \cdot 5^{-k} \cdot k^2}{3k-2} \qquad \lim_{k \to \infty} \frac{a_{k+1}}{a_k}$$

$$\lim_{k \to \infty} \frac{5 \cdot 5^{-(k+1)} \cdot (k+1)^2 \; (3k-2)}{[3 \cdot (k+1) - 2] \quad 5 \cdot 5^{-k} \cdot k^2}$$

$$\frac{5^{-k-1}}{5^{-k}} \cdot \left(\frac{k+1}{k}\right)^2 \cdot \frac{3k-2}{3k+1}$$

$$\downarrow \qquad (1 + 1/k)^2 \qquad \frac{k \cdot (3 - 2/k)}{k \cdot (3 + 1/k)} \quad 2/k \to 0,\; 1/k \to 0 \quad \frac{3^-}{3^+}$$

$$5^{-1} \qquad 1^+ \qquad 1^-$$

$$\downarrow$$

$$1/5 \qquad 1^+ \qquad 1^-$$

$$1/5 < 1 \quad \checkmark$$