$$\sum_{k=0}^{n} 5^k = \frac{5^{n+1}-1}{4}$$

$a_k = 5^k$

$S_n = \frac{5^{n+1}-1}{4}$

n = 0:

$a_0 = S_0$

$1 = 5^0 = \frac{5^{0+1}-1}{4} = 4/4 = 1$ ✓

Prämisse: Es gilt der Zusammenhang $\sum_{k=0}^{n} 5^k = \frac{5^{n+1}-1}{4}$

n+1:

$S_n + a_{n+1} = S_{n+1}$

$\frac{5^{n+1}-1}{4} + 5^{n+1} = \frac{5^{(n+1)+1}-1}{4} \quad | \cdot 4$

$5 \cdot 5^n + 4 \cdot 5 \cdot 5^n = 5^2 \cdot 5^n$

$5^{n+1} - 1 + 4 \cdot 5^{n+1} = 5^{n+2} - 1 \quad | + 1$

$5 \cdot 5^{n+1} = 5^{n+2} = 5^1 \cdot 5^{n+1}$ ✓

$$\sum_{k=1}^{n} k^3 = \left(\frac{n\cdot(n+1)}{2}\right)^2$$

k^3 (!): a_k $\qquad$ $\left(\frac{n\cdot(n+1)}{2}\right)^2$: S_n

$n=1$ $\qquad a_1 = S_1 \iff 1^3 = 1 = \left(\frac{1\cdot(1+1)}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$ ✓

$n+1$: $\qquad \left(\frac{n\cdot(n+1)}{2}\right)^2 + (n+1)^3 = \left[\frac{(n+1)\cdot((n+1)+1)}{2}\right]^2$

$$\tfrac{1}{4}\cdot n^2\cdot(n+1)^2 + (n+1)^3 = \tfrac{1}{4}\cdot(n+1)^2\cdot(n+2)^2 \qquad |\cdot 4$$

$$n^2(n+1)^2 + 4\cdot(n+1)^3 = (n+1)^2\cdot(n+2)^2$$

$$(n+1)^2\left[n^2 + 4\cdot(n+1)\right] = (n+1)^2(n+2)^2 \qquad |:(n+1)^2 \neq 0$$

$$n^2 + 4n + 4 = (n+2)^2 = n^2 + 4n + 4$$

$$0 = 0$$ ✓

1 $\sum_{k=3}^{8} \left(\frac{3}{4}\right)^k = \sum_{k=0}^{8} \left(\frac{3}{4}\right)^k - \left[\left(\frac{3}{4}\right)^0 + \left(\frac{3}{4}\right)^1 + \left(\frac{3}{4}\right)^2\right]$

$\Leftrightarrow |q| < 1$

$= \frac{1-\left(\frac{3}{4}\right)^8}{1-\frac{3}{4}} - \left(1 + \frac{3}{4} + \frac{9}{16}\right)$

$4 \cdot \left(1 - \left(\frac{3}{4}\right)^8\right) - \frac{37}{16}$

2) $\sum_{k=2}^{5} \left(\frac{1}{3}\right)^{k+2} = \sum_{k=2}^{5} \left(\frac{1}{3}\right)^k \cdot \left(\frac{1}{3}\right)^2 = \frac{1}{9} \cdot \sum_{k=2}^{5} \left(\frac{1}{3}\right)^k$

$\frac{1}{9} \cdot \left[\sum_{k=0}^{5} \left(\frac{1}{3}\right)^k - \sum_{k=0}^{1} \left(\frac{1}{3}\right)^k\right]$

$\frac{1}{9} \cdot \left[\frac{1-\left(\frac{1}{3}\right)^6}{2/3} - \frac{1-\left(\frac{1}{3}\right)^2}{2/3}\right]$

$\frac{1}{9} \cdot \frac{3}{2} \cdot \left(1 - \left(\frac{1}{3}\right)^6 - 1 + \frac{1}{9}\right) = \frac{1}{6} \cdot \left(\frac{1}{9} - \left(\frac{1}{3}\right)^6\right)$

3) $\sum_{k=4}^{6} 4 \cdot \left(\frac{1}{2}\right)^{k-2} = 16 \cdot \sum_{k=4}^{6} \left(\frac{1}{2}\right)^{k}$

$\left(\frac{1}{2}\right)^{-2} = 2^2 = 4$

$\Rightarrow 16 \cdot \sum_{k=0}^{2} \left(\frac{1}{2}\right)^{k+4} = 16 \cdot \sum_{k=0}^{2} \left(\frac{1}{2}\right)^{4} \cdot \left(\frac{1}{2}\right)^{k}$

$= 16 \cdot \frac{1}{16} \cdot \frac{1-\left(\frac{1}{2}\right)^3}{1-\frac{1}{2}}$

$= \frac{7/8}{1/2} = \frac{7}{4}$