VOKABELN DER ZAHLENFOLGEN

1) Zahlenfolge
2) Quotientenverfahren
3) Rekursive Definition
4) Infimum
5) Intuitive Definition
6) Konvergenz
7) Monotonie(beweis)
8) Beschränktheit
9) Grenzwert
10) Explizite Definition
11) Differenzmethode
12) Supremum
13) Folgeglied
14) Substitution

1) b) $\quad a_{n+1} = a_n^2 + \frac{1}{4} \quad ; \quad a_1 = 0$

$$a_2 = 0^2 + \frac{1}{4} = \frac{1}{4} = \frac{4}{16}$$

$$a_3 = \left(\frac{1}{4}\right)^2 + \frac{1}{4} = \frac{1}{16} + \frac{4}{16} = \frac{5}{16}$$

Monotonie: Behauptung $a_{n+1} > a_n$

$n = 1 \qquad a_2 > a_1 \qquad \frac{1}{4} > 0 \qquad$ ✓

$n+1: \qquad a_{n+2} > a_{n+1}$

$$a_{n+1}^2 + \frac{1}{4} > a_n^2 + \frac{1}{4} \qquad \left| -\frac{1}{4}\right.$$

$$a_{n+1}^2 > a_n^2 \qquad \left| \sqrt{}\right.$$

$$a_{n+1} > a_n \qquad ✓$$

Schranke: $a_1 = 0$ ist untere Schranke

Behauptung $a_n < 1/2$

$n = 1$ $\quad a_1 = 0 < 1/2$ ✓

$n+1$: $\quad a_n < 1/2 \quad |(\ )^2 \rightarrow a_n^2 + 1/4$

$$a_n^2 < 1/4 \quad | + 1/4$$

$$a_n^2 + 1/4 < 1/4 + 1/4 = 1/2$$ ✓

<u>Grenzwert.</u>

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} a_n$$

$$\lim_{n\to\infty} a_n = r$$

$$a_n^2 + 1/4 = a_n$$

$$r^2 + 1/4 = r \qquad 1-r$$

$$r^2 - r + 1/4 = 0$$

$$(r - 1/2)^2 = 0 \qquad r = 1/2$$

$$r_{1,2} = 1/2 \pm \sqrt{(1/2)^2 - 1/4}$$

$a_k = 2k - 1 \quad ; k \in \mathbb{N}^{\geq 1}$

↳ ungerade Zahlen

$a_5 = 2 \cdot 5 - 1 = 10 - 1 = 9$ -> Folgeglied an Pos. 5

Summe alle ungeraden Zahlen bis zur Pos 5

↳ $1 + 3 + 5 + 7 + 9 = 25 = \sum_{k=1}^{5} (2k-1)$

$$\sum_{k=1}^{n} \underbrace{(2k-1)}_{a_k}$$

$\sum$: Summe

$\prod$: Produkt

$$\sum_{k=1}^{n} \underbrace{(2k-1)}_{a_k} = n^2$$

$n=1$: 1

$n=2$: $1+3=4$

$n=3$: $1+3+5=9$

$n=1$: $\sum_{k=1}^{1}(2k-1) = 1^2$

$$1 = 1$$

$n+1$: $\sum_{k=1}^{n+1}(2k-1) = (n+1)^2$

$$\underbrace{a_1+a_2+a_3+\dots+a_n}_{n^2} + a_{n+1}$$

$$n^2 + 2\cdot(n+1)-1 = (n+1)^2$$

$$n^2+2n+2-1 = n^2+2n+1$$

$$0 = 0$$

3) $$\sum_{k=1}^{n} \underbrace{\left(\frac{k}{2^k}\right)}_{a_k} = \underbrace{2 - \frac{n+2}{2^n}}_{S_n}$$

$n = 1$ $\qquad a_1 = S_1$

$$\frac{1}{2^1} = \frac{1}{2} = 2 - \frac{1+2}{2} = 2 - \frac{3}{2} = \frac{1}{2} \quad ✓$$

$n+1$ $\qquad S_n + a_{n+1} = S_{n+1}$

$$2 - \frac{n+2}{2^n} + \frac{n+1}{2^{n+1}} = 2 - \frac{(n+1)+2}{2^{n+1}} \quad | -2$$

$$\frac{-2\cdot(n+2) + n+1}{2^{n+1}} = -\frac{n+3}{2^{n+1}} \quad | \cdot 2^{n+1}$$

$$-2n - 4 + n + 1 = -n - 3 \quad | +n \; +3$$

$$0 = 0 \quad ✓$$