

5.106

1) a) $y'' = 8 \cdot e^{2x+1} + 2$; $y(0) = e$; $y(1) = 2e^3 - e$
 $y' = 4 \cdot e^{2x+1} + 2x + C_1$
 $y = 2e^{2x+1} + x^2 + C_1 x + C_2$

$$y(0) = 2e + C_2 = e \Rightarrow C_2 = -e$$

$$y(1) = 2e^3 + 1 + C_1 \cdot 1 - e = 2e^3 - e \Rightarrow C_1 = -1$$
$$\underline{y = 2e^{2x+1} + x^2 - x - e} \quad \text{spezielle Lösung}$$

5) $y' = \frac{3}{(1-x)^2} = 3 \cdot (1-x)^{-2}$; $y(2) = 5$

$$y = 3 \cdot (1-x)^{-1} + C_1 = \frac{3}{(1-x)} + C_1$$

$$y(2) = -3 + C_1 = 5 \Rightarrow C_1 = 8$$

$$\underline{y = \frac{3}{1-x} + 8} \quad \text{spezielle Lösung}$$

2) a) $x(t) = \sin(2t - 3)$

$$x'(t) = 2 \cdot \cos(2t - 3)$$

$$x''(t) = -4 \cdot \sin(2t - 3) = -4 \cdot x(t)$$

$$x''(t) + 4 \cdot x(t) = 0$$

homogene DGL

5) $y(x) = \frac{1}{10} e^{2-5x}$

$$y'(x) = -\frac{1}{2} e^{2-5x}$$

$$y''(x) = \frac{5}{2} e^{2-5x} = 25 \cdot y(x)$$

$$y''(x) - 25y(x) = 0$$

homogene DGL



SM10

1) a) $y' = \frac{dy}{dx} = (y+1) \cdot \sin(x) ; y(\pi/2) = 4$

$$\frac{dy}{y+1} = \sin(x) dx \quad | \int$$

$$\ln(y+1) = -\cos(x) + \ln(C_1) \quad | -\ln(C_1)$$

$$\ln(y+1) - \ln(C_1) = \ln\left(\frac{y+1}{C_1}\right) = -\cos(x)$$

$$\frac{y+1}{C_1} = e^{-\cos(x)} \Rightarrow y = C_1 \cdot e^{-\cos(x)} - 1$$

$$y(\pi/2) = C_1 \cdot e^0 - 1 = 4 \Rightarrow C_1 = 5$$

$$y = 5 \cdot e^{-\cos(x)} - 1 \quad \text{spezielle Lösung}$$

5) $x + y \cdot y' = x + y \frac{dy}{dx} = 0 ; y(0) = 2$

$$y dy = -x dx \quad | \int$$

$$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C$$

$$y = \sqrt{2C_1 - x^2}$$

$$y(0) = \sqrt{2C_1 - 0} = 2 \Rightarrow C_1 = 2$$

$$y = \sqrt{4 - x^2} \quad \text{spezielle Lösung}$$

2) $10 \cdot \frac{dv}{dt} + v = 40 ; v(0) = 10$

$$\frac{dv}{v-40} = -\frac{1}{10} dt \quad | \int$$

$$\ln(v-40) = -\frac{1}{10}t + \ln(C_1)$$

$$v = C_1 \cdot e^{-\frac{1}{10}t} + 40$$

$$v(0) = C_1 + 40 = 10 \Rightarrow C_1 = -30$$

$$v = -30e^{-\frac{1}{10}t} + 40 \quad \text{spezielle Lösung}$$

$$\lim_{t \rightarrow \infty} [-30e^{-\frac{1}{10}t} + 40] = 40 \quad \text{Endgeschwindigkeit}$$



5.116

a) $x^2 y' = y^2 - xy \quad ; \quad y(-1) = 1 \quad \wedge \quad y' = f\left(\frac{y}{x}\right)$

$$y' = \left(\frac{y}{x}\right)^2 - \frac{y}{x}$$

$$\mu = \frac{y}{x}$$

$$\underbrace{\mu' \cdot x + \mu}_{\mu' \cdot x + \mu} = \mu^2 - \mu$$

$$y = k \cdot x \Rightarrow y' = \mu' \cdot x + \mu$$

$$\mu' \cdot x = \mu^2 - 2\mu$$

$$\frac{d\mu}{\mu^2 - 2\mu} = \frac{dx}{x}$$

$$\int \frac{1}{x^2 - 2x} dx = \frac{1}{2} \ln\left(\frac{x-2}{x}\right)$$

$$\frac{1}{2} \cdot \ln\left(\frac{\mu-2}{\mu}\right) = \ln(x) + \ln(C_1)$$

$$\mu = \frac{2}{1 - C_1 \cdot x^2} = \frac{2}{1 - C_2 \cdot x^2} \quad \wedge \quad y = \mu \cdot x$$

$$y = \frac{2x}{1 - C_2 x^2}$$

$$y(-1) = \frac{-2}{1 - C_2} = 1 \Rightarrow C_2 = 3$$

$$y = \frac{2x}{1 - 3x^2}$$

spezielle Lösung

5) $(1+x^2) y y' = x \cdot (1+y^2), \quad y(1) = 3$

$$(1+x^2) \cdot \frac{1}{2} \mu' = x \cdot \mu \quad \mu = 1+y^2$$

$$\frac{d\mu}{\mu} = \frac{2x}{1+x^2} dx \quad \mu' = 2 \cdot y \cdot y'$$

$$\ln(\mu) = \ln(1+x^2) + \ln(C_1) = \ln[C_1 \cdot (1+x^2)]$$

$$\mu = C_1 \cdot (1+x^2) = 1+y^2$$

$$y = \sqrt{C_1(1+x^2) - 1}$$

$$y(1) = \sqrt{2C_1 - 1} = 3 \Rightarrow C_1 = 5$$

$$y = \sqrt{5x^2 + 4}$$

spezielle Lösung



a) $y' \cdot \sin(x) - y \cdot \cos(x) = 4 \cdot \sin^4(x)$

homogene Form ($=0$) $g(x)$

$$\frac{dy}{y} = \frac{\cos(x)}{\sin(x)} dx = \cot(x)$$

$$\ln(y) = \ln(\sin(x)) + \ln(C_1)$$

Variation $\begin{cases} \Rightarrow y = K \cdot \sin(x) \\ \Rightarrow y = K(x) \cdot \sin(x) \end{cases}$ allgemeine Lösung

$$y' = K'(x) \cdot \sin(x) + K(x) \cdot \cos(x)$$

Einsetzen in die Aufgabenstellung

$$\left[K'(x) \cdot \sin(x) + K(x) \cdot \cos(x) \right] \cdot \sin(x) - \left(K(x) \cdot \sin(x) \right) \cdot \cos(x) = 4 \cdot \sin^4(x)$$

$$K'(x) \cdot \sin^2(x) = 4 \cdot \sin^4(x)$$

$$K'(x) = 4 \cdot \sin^2(x)$$

$$\int \sin^2(x) dx = \frac{x}{2} - \frac{\sin(2x)}{4}$$

Einsetzen
in die
Variation $\begin{cases} K(x) = 2x - \sin(2x) + C_1 \\ y = (2x - \sin(2x) + C_1) \cdot \sin(x) \end{cases}$

5) $y' + \frac{y}{x} = \cos(x)$
 $=0$ $g(x)$

$$\frac{dy}{y} = -\frac{dx}{x} \Rightarrow \ln(y) = -\ln(x) + \ln(C_1) = \ln\left(\frac{C_1}{x}\right)$$

$$y = \frac{C_1}{x} = \frac{K(x)}{x}$$

$y = \frac{\cos(x) + x \cdot \sin(x) + C}{x}$

$y' = \frac{K'(x) \cdot x - K(x)}{x^2}$

$\frac{K'(x)}{x} - \frac{K(x)}{x^2} + \frac{K(x)}{x^2} = \cos(x)$

$K'(x) = x \cdot \cos(x) \rightarrow \text{partielle Integration}$

$K(x) = \cos(x) + x \cdot \sin(x) + C$



5.127

a) $y' + 5y = -26 \sin(x)$; $y(0) = 0$

$\underbrace{y'}_0 + \underbrace{5y}_{g(x)} = -26 \sin(x)$

$$\frac{dy}{y} = -5 dx \Leftrightarrow \ln(y) = -5x + \ln(C_1)$$

$$\Rightarrow y = C_1 \cdot e^{-5x}$$

partikuläre Lösung

$$y_p = \underline{C_1 \cdot \sin(x)} + \underline{C_2 \cdot \cos(x)}$$

$$y'_p = \underline{C_1 \cdot \cos(x)} - \underline{C_2 \cdot \sin(x)}$$

Ersetzen in $y'_p + 5y_p$: $\underline{[C_1 \cdot \cos(x) - C_2 \cdot \sin(x)]} + 5 \cdot \underline{[C_1 \cdot \sin(x) + C_2 \cdot \cos(x)]} = (5C_2 + C_1) \cdot \cos(x) + (5C_1 - C_2) \cdot \sin(x) = -26 \cdot \sin(x)$

$$5C_2 + C_1 = 0 \quad \wedge \quad 5C_1 - C_2 = -26$$

$$C_1 = -5C_2 \quad \wedge \quad 5 \cdot (-5C_2) - C_2 = -26 \Rightarrow C_2 = 1$$

$$\Rightarrow y_p = -5 \cdot \sin(x) + 1 \cdot \cos(x)$$

$y = y_0 + y_p$: $-5 \cdot \sin(x) + \cos(x) + \underbrace{C_1 \cdot e^{-5x}}_0$

$$y(0) = -5 \cdot \sin(0) + \cos(0) + \underbrace{C_1 \cdot 1}_0 = 0 = 0$$

$$C_1 = -1$$

$$y = -5 \sin(x) + \cos(x) - e^{-5x}$$

spezielle Lösung



$$5) \quad \underbrace{y' + 3y}_{g} = \underbrace{e^x + 2 \cdot \cos(2x)}_{g(x)}$$

$$\frac{dy}{y} = -3 dx \Leftrightarrow \ln(y) = -3x + \ln(C_1)$$

$$y = C_1 \cdot e^{-3x} \Rightarrow K(x) \cdot e^{-3x} \quad \text{Variation}$$

$$y' = K'(x) \cdot e^{-3x} - 3K(x) e^{-3x}$$

$$\text{Einsetzen in } y' + 3y \quad [K'(x) \cdot e^{-3x} - 3K(x)e^{-3x}] + 3 \cdot K(x)e^{-3x} = e^x + 2 \cdot \cos(2x)$$

$$K'(x) \cdot e^{-3x} = e^x + 2 \cdot \cos(2x)$$

$$K'(x) = e^{4x} + 2e^{3x} \cdot \cos(2x)$$

partielle Integration

$$y = K(x) \cdot e^{-3x} \quad K(x) = \frac{1}{4}e^{4x} + \frac{2}{13}e^{3x} \cdot (3\cos(2x) + 2 \cdot \sin(2x)) + C$$

$$\Rightarrow y = \frac{1}{4}e^x + \frac{2}{13} \cdot (3\cos(2x) + 2 \cdot \sin(2x)) + C$$

