Musterlösung Vorkursklausur 2020

1) Menge A: $A = \{1; 3; 4; 6; 7; 9; 10; 11; 12; 13; 16\}$
   Menge B: $B = \{2; 4; 5; 6; 8; 10; 12; 14; 15; 16\}$

   a) $A \cap B$: $\{4; 6; 10; 12; 16\}$
   b) $A \cup B$: $\{x \in \mathbb{N} | x \geq 1 \wedge x \leq 16\}$
   c) $A \backslash B$: $\{x \in \mathbb{N} \backslash \{5\} | x mod 2 <> 0 \wedge x \leq 13\} = \{1; 3; 7; 9; 11; 13\}$
   d) $B \backslash A$: $\{2; 5; 8; 14; 15\}$

2) a) $2 + \frac{1}{x} - \frac{2}{3} - \frac{1}{4x} - \left(\frac{7}{12} + \frac{3}{4}\right) = \frac{24x + 12 - 8x - 3 - 7x - 9x}{12x} = \frac{9}{12x} = \frac{3}{4x}$

   b) $\frac{\frac{3a}{b} - 2 + \frac{b}{3a}}{\frac{18a}{b} - \frac{2b}{a}} = \frac{\frac{9a^2 - 6ab + b^2}{3ab}}{\frac{18a^2 - 2b^2}{ab}} = \frac{(3a-b)^2}{3ab} \cdot \frac{ab}{2 \cdot (3a-b) \cdot (3a+b)} = \frac{3a - b}{18a + 6b}$

3) a) $\left(\frac{4}{x^2}\right)^2 \cdot (xy^2 + 0{,}5x)^4 - 8y^2 \cdot (1 + 3y^2 + 4y^4) = \frac{16}{x^4} \cdot \left(x^4y^8 + 2x^4y^6 + \frac{3}{2}x^4y^4 + \frac{1}{2}x^4y^2 + \frac{1}{16}x^4\right) - 8y^2 - 24y^4 - 32y^6$

   $16y^8 + 32y^6 + 24y^4 + 8y^2 + 1 - 8y^2 - 24y^4 - 32y^6 = 16y^8 + 1$

   b) $-6 \cdot (-3x - 2 \cdot (y - (4x + y - 3 \cdot (2x - y)) + z - x) - 2 \cdot (3y + z))$
   $-6 \cdot (-3x - 2 \cdot (y - 4x - y + 6x - 3y + z - x) - 6y - 2z) = -6 \cdot (-3x - 2 \cdot (x - 3y + z) - 6y - 2z) = 30x + 24z$

4) a) $z = 2 \cdot \frac{5i}{3i - 4} - \frac{6i - 4}{2 - i} + \frac{1}{5}i = \frac{10i}{3i - 4} \cdot \frac{3i + 4}{3i + 4} - \frac{6i - 4}{2 - i} \cdot \frac{2 + i}{2 + i} + \frac{i}{5} = \frac{10i \cdot (3i + 4)}{(3i)^2 - 4^2} - \frac{(6i - 4) \cdot (2 + i)}{2^2 - i^2} + \frac{i}{5}$

   $z = \frac{-30 + 40i}{-25} - \frac{12i + 6i^2 - 8 - 4i}{5} + \frac{i}{5} = \frac{5 \cdot (-6 + 8i)}{-25} - \frac{8i - 14}{5} + \frac{i}{5} = \frac{-(-6 + 8i) - (8i - 14) + i}{5}$

   $z = \frac{6 - 8i - 8i + 14 + i}{5} = \frac{20 - 15i}{5} = \frac{20}{5} - \frac{15}{5}i = 4 - 3i$

   $\alpha = \arctan\left(-\frac{3}{4}\right) + 2\pi \approx 323°$

   $r = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5$

b) $z^3+(6-2i)\cdot z^2-2i\cdot z\cdot(3-4i)=0$

$$z\cdot\left[z^2+(6-2i)\cdot z+(-8-6i)\right]=0\Rightarrow z_1=0\vee z^2+(6-2i)\cdot z+(-8-6i)=0$$
$$z_{2;3}=-\frac{6-2i}{2}\pm\sqrt{\left(\frac{6-2i}{2}\right)^2-(-8-6i)}=-(3-i)\pm\sqrt{(3-i)^2+8+6i}$$
$$z_{2;3}=-3+i\pm\sqrt{9-6i+i^2+8+6i}=-3+i\pm\sqrt{16}\Rightarrow z_2=1+i\vee z_3=-7+i$$
$$z_1=0\wedge r_1=0\wedge\alpha_1=0°$$
$$z_2=1+i\wedge r_2=\sqrt{2}\wedge\alpha_2=\arctan(1)$$
$$z_3=-7+i\wedge r_3=\sqrt{50}\wedge\alpha_3=\arctan\left(-\frac{1}{7}\right)+\pi$$

5) a) $\frac{9\cdot(0{,}5\cdot x^2y^{-2}z)^4}{54\cdot(4\cdot x^{-2}y^3z^{-2})^{-3}}:\frac{36\cdot(2\cdot x^2y^5z^{-4})^2}{16\cdot(3\cdot x^4y^3z^{-4})^3}=\frac{3^2\cdot2^{-4}\cdot x^8y^{-8}z^4}{3^3\cdot2\cdot2^{-6}\cdot x^6y^{-9}z^6}\cdot\frac{2^4\cdot3^3\cdot x^{12}y^9z^{-12}}{3^2\cdot2^2\cdot2^2\cdot x^4y^{10}z^{-8}}=\frac{2\cdot x^{10}}{z^6}$

b) $4^{ld3}-\left(\frac{1}{\sqrt{e}}\right)^{4\cdot\ln0{,}25}+4\cdot\log\sqrt{1000}-\frac{1}{2}\cdot ld64-0{,}01^{\log0{,}5}+12\ln\sqrt[3]{e^2}$

$$2^{2\cdot ld3}-e^{-\frac{1}{2}\cdot4\ln0{,}25}+\log\left(10^{\frac{3}{2}}\right)^4-ld\left(2^6\right)^{\frac{1}{2}}-10^{-2\log0{,}5}+\ln\left(e^{\frac{2}{3}}\right)^{12}=9-16+6-3-4+8=0$$

c) $\frac{\sqrt[2n]{a^{3n+7}}}{\sqrt{\sqrt[n]{a^{5-2n}}}}\cdot\left(\sqrt[4n]{a^2}\right)^{5n-2}=\frac{a^{\frac{3n+7}{2n}}}{a^{\frac{5-2n}{2n}}}\cdot a^{\frac{5n-2}{2n}}=a^{\frac{3n+7-(5-2n)+5n-2}{2n}}=a^{\frac{10n}{2n}}=a^5$

d) $3\,log\,x-2\,log\frac{2}{x^2}-3\,log\,4+4\,log\sqrt{x}=\frac{1}{2}\cdot\left(log\,x^4-\frac{1}{2}log\,256\right)-\frac{1}{3}log\frac{1}{8}$

$$log\,x^3-log\frac{4}{x^4}-log\,4^3+log\,x^2=log\,x^2-log\,256^{\frac{1}{4}}-log\frac{1}{2}\Leftrightarrow log\frac{x^3\cdot x^2}{\frac{4}{x^4}\cdot4^3}=log\frac{x^2}{4\cdot\frac{1}{2}}\Leftrightarrow x^7=128\Leftrightarrow x=\sqrt[7]{128}=2$$

6) a) $f(x)=-0{,}5\cdot x^2+6\cdot x-16=-0{,}5\cdot(x^2-12x+32)=-0{,}5\cdot(x-8)\cdot(x-4)$

Achsenschnittpunkte: $S_y=(0/-16)\vee S_{x_1}(8/0)\vee S_{x_2}(4/0)$

Scheitelpunkt: $S=(6/f(6))=(6/2)$ ist ein Hochpunkt, da -0,5 > 0

Verlauf: Die Parabel ist nach unten geöffnet, da -0,5 > 0
Die Parabel verläuft flacher, da |-0,5| < 1

b) $f(x)=3\cdot x^2+18\cdot x-81=3\cdot(x^2+6x-27)=3\cdot(x+9)\cdot(x-3)$

Achsenschnittpunkte: $S_y=(0/-81)\vee S_{x_1}(-9/0)\vee S_{x_2}(3/0)$

Scheitelpunkt: $S=(-3/f(-3))=(-3/108)$ ist ein Tiefpunkt, da 3 > 0

Verlauf: Die Parabel ist nach oben geöffnet, da 3 > 0
Die Parabel verläuft steiler, da |3| > 1

7) a) $\frac{36x-108}{x-3} = x^3+3x^2-16x-12 \Rightarrow D = R/\{3\}$

$36x-108 = (x^3+3x^2-16x-12)\cdot(x-3) \Leftrightarrow 36x-108 = x^4+3x^3-16x^2-12x-3x^3-9x^2+48x+36$

$x^4-25x^2+144 = (x^2-16)\cdot(x^2-9) = (x-4)\cdot(x+4)\cdot(x-3)\cdot(x+3) = 0$

$\Rightarrow L = \{-4;-3;4\}$

b) $x^3+5x = 6\cdot(x^2-2) \Leftrightarrow x^3-6x^2+5x+12$

$(x^3-6x^2+5x+12):(x+1) = x^2-7x+12$

$x^3-6x^2+5x+12 = (x+1)\cdot(x-3)\cdot(x-4) = 0 \Rightarrow L = \{-1;3;4\}$

8) a)

$x \geq 3$	$x < 3$
$-9+3x > 2x+4 \Leftrightarrow x > 13$	$9-3x > 2x+4 \Leftrightarrow -5x > -5 \Leftrightarrow x < 1$
$x > 13$	$x < 1$
Probe: $x = 14 \Rightarrow \lvert 9-42 \rvert = 33 > 32$	Probe: $x = 0 \Rightarrow \lvert 9-0 \rvert = 9 > 4$

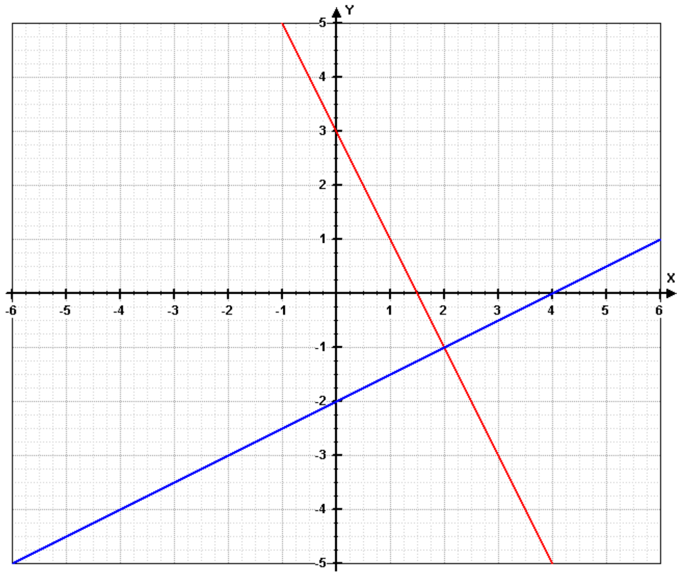
$$L = \{x \in R \mid x > 13 \vee x < 1\}$$

b) $4\cdot\frac{5-x^2}{6-2x} \geq 2x+5$

$x < 3$	$x > 3$
$20-4x^2 \geq (2x+5)\cdot(6-2x) = 12x-4x^2+30-10x$ $20 \geq 2x+30 \Leftrightarrow x \leq -5$	$20-4x^2 \leq (2x+5)\cdot(6-2x) = 12x-4x^2+30-10x$ $20 \leq 2x+30 \Leftrightarrow x \geq -5$
$x \leq -5$	$x > 3$
Probe: $x = -7 \Rightarrow 4\cdot\frac{5-49}{6+14} = \frac{-44}{5} \geq -14+5 = -9$	Probe: $x = 4 \Rightarrow 4\cdot\frac{5-16}{6-8} = \frac{-44}{-2} \geq 8+5 = 13$

$$L = \{x \in R \mid x > 3 \vee x \leq -5\}$$

9) a) $\begin{vmatrix} 2x+y=3 \\ 2y+4=x \end{vmatrix} \Rightarrow y=-2x+3 \wedge y=\frac{1}{2}x-2 \Rightarrow S(2/-1)$



b) Gauß-Verfahren:

$$\left|\begin{array}{ccccccc} -x & + & 2\cdot y & - & z & = & -5 \\ x & - & 3\cdot y & + & 2\cdot z & = & 8 \\ 2\cdot x & - & y & + & 5\cdot z & = & 13 \end{array}\right| \quad |\cdot 1 \quad |\cdot(2)$$

$$\Leftrightarrow \left|\begin{array}{ccccccc} -x & + & 2\cdot y & - & z & = & -5 \\ 0 & - & y & + & z & = & 3 \\ 0 & + & 3\cdot y & + & 3\cdot z & = & 3 \end{array}\right| \quad |\cdot 3$$

$$\Leftrightarrow \left|\begin{array}{ccccccc} -x & + & 2\cdot y & - & z & = & -5 \\ 0 & - & y & + & z & = & 3 \\ 0 & + & 0 & + & 6\cdot z & = & 12 \end{array}\right| \Rightarrow L=\{(1;-1;2)\}$$

c) Gleichsetzungsverfahren:

$$\left|\begin{matrix} \frac{2}{5}x-4y=-4 \\ 6x-24y=12 \end{matrix}\right| \Rightarrow x=10y-10 \wedge x=4y+2$$

$$10y-10=4y+2 \Leftrightarrow 6y=12 \Leftrightarrow y=2 \Rightarrow x=10 \Rightarrow L=\left(\left(10;2\right)\right)$$

10) $g_1: \vec{x} = \begin{pmatrix} -1 \\ 4 \\ -7 \end{pmatrix} + \propto \cdot \begin{pmatrix} 2+1 \\ -5-4 \\ 8+7 \end{pmatrix} = \begin{pmatrix} -1 \\ 4 \\ -7 \end{pmatrix} + \propto \cdot \begin{pmatrix} 3 \\ -9 \\ 15 \end{pmatrix}$

$$g_2: \vec{x} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \beta \cdot \begin{pmatrix} 3-1 \\ -5+1 \\ 7-1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \beta \cdot \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} 3 \\ -9 \\ 15 \end{pmatrix} = \gamma \cdot \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} \to \gamma = \frac{3}{2} \wedge \gamma = \frac{9}{4} \wedge \gamma = \frac{15}{6}$$

Die beiden Richtungsvektoren sind linear unabhängig.

$$\begin{pmatrix} -1 \\ 4 \\ -7 \end{pmatrix} + \propto \cdot \begin{pmatrix} 3 \\ -9 \\ 15 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \beta \cdot \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix} \Leftrightarrow \propto \cdot \begin{pmatrix} 3 \\ -9 \\ 15 \end{pmatrix} + \beta \cdot \begin{pmatrix} -2 \\ 4 \\ -6 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 8 \end{pmatrix}$$

$$\left|\begin{matrix} 3\alpha & - & 2\beta & = & 2 \\ -9\alpha & + & 4\beta & = & -5 \\ 15\alpha & - & 6\beta & = & 8 \end{matrix}\right| \Leftrightarrow \left|\begin{matrix} 3\alpha & - & 2\beta & = & 2 \\ 0 & - & 2\beta & = & 1 \\ 0 & + & 4\beta & = & -2 \end{matrix}\right| \Rightarrow \beta = -\frac{1}{2} \wedge \alpha = \frac{1}{3}$$

Die beiden Geraden schneiden sich im Punkt $(0; 1; -2)$

11) $f(x) = 3 \cdot \sin(0{,}25x - 9{,}5\pi) - 5$

Additionstheorem:

$$\sin\left(\frac{1}{4}x - \frac{19}{2}\pi\right) = \sin\left(\frac{1}{4}x\right) \cdot \cos\left(\frac{19}{2}\pi\right) - \cos\left(\frac{1}{4}x\right) \cdot \sin\left(\frac{19}{2}\pi\right) = \cos\left(\frac{1}{4}x\right)$$

$$\Rightarrow f(x) = 3 \cdot \cos\left(\frac{1}{4}x\right) - 5$$

Wertebereich:

$$3 \cdot [-1;1] - 5 = [-3;3] - 5 \Rightarrow W = y \in [-8;-2]$$

Achsensymmetrie zur y-Achse:

$$f(x) = f(-x)$$

$$3 \cdot \cos\left(\frac{1}{4}x\right) - 5 = 3 \cdot \cos\left(-\frac{1}{4}x\right) - 5$$
$$\cos\left(\frac{1}{4}x\right) = \cos\left(-\frac{1}{4}x\right)$$

Periode: $P = 2\pi \cdot \frac{4}{1} = 8\pi$

$$f(x) = f(x + 8\pi)$$

$$3 \cdot \cos\left(\frac{1}{4}x\right) - 5 = 3 \cdot \cos\left(\frac{1}{4}(x + 8\pi)\right) - 5$$

$$\cos\left(\frac{1}{4}x\right) = \cos\left(\frac{1}{4}x + 2\pi\right) = \cos\left(\frac{1}{4}x\right) \cdot \cos(2\pi) - \sin\left(\frac{1}{4}x\right) \cdot \sin(2\pi) = \cos\left(\frac{1}{4}x\right) \cdot 1 - \sin\left(\frac{1}{4}x\right) \cdot 0 = \cos\left(\frac{1}{4}x\right)$$