S. 130 Nr. 1 $\qquad A \cdot \vec{x} = \lambda \cdot \vec{x}$

$$\begin{pmatrix} 1 & 0 & 0 \\ -1 & 3 & 0 \\ 0 & -3 & 0 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 1 \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ -x_1 + 3x_2 \\ -3x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \Rightarrow \left| \begin{array}{rcl} x_1 & = & x_1 \\ -x_1 + 3x_2 & = & x_2 \\ -3x_2 & = & x_3 \end{array} \right|$$

$$\left| \begin{array}{rcl} 0 & = & 0 \\ -x_1 + 2x_2 & = & 0 \\ -3x_2 - x_3 & = & 0 \end{array} \right| \qquad x_2 = \alpha \qquad \left| \begin{array}{rcl} 0 & = & 0 \\ -x_1 + 2\alpha & = & 0 \\ -3\alpha - x_3 & = & 0 \end{array} \right| \begin{array}{l} x_1 = 2\alpha \\ x_3 = -3\alpha \end{array}$$

$$\vec{x} = \begin{pmatrix} 2\alpha \\ \alpha \\ -3\alpha \end{pmatrix}$$

2)

$$\det\begin{pmatrix} 1-\lambda & \boxed{2} & 3 \\ \boxed{2} & -4-\lambda & \boxed{-2} \\ 3 & \boxed{-2} & 1-\lambda \end{pmatrix} +$$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 2 & -4-\lambda & -2 \\ 4-\lambda & 0 & 4-\lambda \end{vmatrix}$$

$+$

$$\begin{vmatrix} 4-\lambda & 2 & 3 \\ 0 & -4-\lambda & -2 \\ 8-2\lambda & 0 & 4-\lambda \end{vmatrix} = (4-\lambda)^2\cdot(-4-\lambda)-4\cdot(8-2\lambda) - 3\cdot(-4-\lambda)(8-2\lambda)$$

$$-(4-\lambda)^2\cdot(4+\lambda)-8(4-\lambda)-3(4+\lambda)\cdot 2(4-\lambda)$$

$$(4-\lambda)\cdot\left[-(4-\lambda)(4+\lambda)-8-6(4+\lambda)\right]$$

$$(4-\lambda)\cdot\left[-(4-\lambda)(4+\lambda)-8-6(4+\lambda)\right]=0$$

$$(4-\lambda)\cdot\left[-(16-\lambda^2)-8-24-6\lambda\right]=0$$

$$(4-\lambda)\left[\lambda^2-6\lambda-48\right]=0$$

$$\lambda_{1/2}=3\pm\sqrt{9+48}=3\pm\sqrt{57}$$

$$\lambda_3=4$$