Trigonometrie

Additionstheoreme:

SIN: $\sin(\alpha \pm \beta) = \sin\alpha \cdot \cos\beta \pm \sin\beta \cdot \cos\alpha$

COS: $\cos(\alpha \pm \beta) = \cos\alpha \cdot \cos\beta \mp \sin\alpha \cdot \sin\beta$

$$f(x) = a \cdot \sin(bx + c) + d$$

$a \;\hat{=}$ Amplitudenmodulation

$b \;\hat{=}$ Periode $\left(P_{NEU} = \frac{P_{ALT}}{b} \right)$

$c \;\hat{=}$ Phasenverschiebung → x-Richtung

$d \;\hat{=}$ Funktionsverschiebung in y-Richtung

$$f(x) = \tfrac{1}{2} \cdot \cos(3x + 3\pi) - 7$$

Phasenverschiebung von 3π

3π

$$\cos(3x + 3\pi) = \underbrace{\cos(3x) \cdot \underbrace{\cos(3\pi)}_{-1}}_{-\cos 3x} - \underbrace{\sin 3x \cdot \underbrace{\sin 3\pi}_{0}}_{0}$$

$$f(x) = \tfrac{1}{2} \cdot (-\cos 3x) - 7 = -\tfrac{1}{2} \cdot \cos(3x) - 7$$

Wertebereich

$$-\tfrac{1}{2} \cdot [-1; 1] - 7$$

$$[-\tfrac{1}{2}; \tfrac{1}{2}] - 7$$

$$\mathbb{W} = y \in [-7{,}5; -6{,}5]$$

Periode $P_{neu} = \frac{2\pi}{3} = \frac{2}{3}\pi$ $f(x) = f\left(x + \frac{2}{3}\pi\right)$

$$f(x) = -\frac{1}{2} \cos(3x) - 7$$

$$\updownarrow =$$

$$f\left(x + \frac{2}{3}\pi\right) = -\frac{1}{2} \cdot \cos\left[3 \cdot \left(x + \frac{2}{3}\pi\right)\right] - 7$$

$$\cos(3x + 2\pi) = \cos 3x \cdot \underset{1}{\underset{\downarrow}{\cos 2\pi}} - \sin 3x \cdot \underset{0}{\underset{\downarrow}{\sin 2\pi}}$$

$$= \cos 3x$$

$\cos(x) = \cos(-x)$

Symmetrie: $f(x) = f(-x)$

$$-\frac{1}{2} \cos 3x - 7 = -\frac{1}{2} \cos(-3x) - 7 \quad | +7 \cdot (-2)$$

$3x = \alpha$

$$\cos(3x) = \cos(-3x)$$

$$\cos(\alpha) = \cos(-\alpha)$$

1) $f(x) = -2 \cdot \sin\left(\frac{1}{3}x - \frac{3}{2}\pi\right) + 5$

2) $|\sin(2x)| = |\cos(2x)|$; $x \in [-180°; 180°]$

1) • Phasenverschiebung von $-270°$

$$\sin\left(\tfrac{1}{3}x - \tfrac{3}{2}\pi\right) = \underbrace{\sin\left(\tfrac{1}{3}x\right) \cdot \underset{0}{\underset{\downarrow}{\cos\left(\tfrac{3}{2}\pi\right)}}}_{0} - \underbrace{\cos\left(\tfrac{1}{3}x\right) \cdot \underbrace{\sin\left(\tfrac{3}{2}\pi\right)}_{-1}}_{\cos\left(\frac{1}{3}x\right)}$$

$$\Rightarrow f(x) = -2 \cdot \cos\left(\tfrac{1}{3}x\right) + 5$$

• Wertebereich: $-2\,[-1; 1] + 5 = [-2; 2] + 5$

$y \in [3; 7]$

• Periode : $p_{neu} = \dfrac{2\pi}{1/3} = 6\pi$

$$f(x) = f(x + 6\pi)$$

$$f(x+6\pi) = -2 \cdot \underbrace{\cos\left(\tfrac{1}{3}(x+6\pi)\right)} + 5$$

$$\cos\left(\tfrac{1}{3}x + 2\pi\right) = \left[\cos \tfrac{1}{3}x \cdot \underbrace{\cos 2\pi}_{1} - \sin \tfrac{1}{3}x \cdot \underbrace{\sin 2\pi}_{0}\right] = \cos\left(\tfrac{1}{3}x\right)$$

$$f(x+6\pi) = -2 \cdot \cos\left(\tfrac{1}{3}x\right) + 5 = f(x)$$

• Symmetrie : $f(x) = f(-x)$

$$-2 \cdot \cos\left(\tfrac{1}{3}x\right) + 5 = -2\cos\left(-\tfrac{1}{3}x\right) + 5 \quad \left| -5 \cdot \left(-\tfrac{1}{2}\right)\right.$$

$\tfrac{1}{3}x = \alpha$

$$\cos\left(\tfrac{1}{3}x\right) = \cos\left(-\tfrac{1}{3}x\right)$$

$$\cos(\alpha) = \cos(-\alpha) \quad \checkmark$$