1) $2 + \frac{1}{x} - \frac{2}{3} - \frac{1}{4x} - \left(\frac{7}{12} + \frac{3}{4}\right) \to \frac{3}{4x}$

2) $$\frac{\frac{3a}{b} - 2 + \frac{b}{3a}}{\frac{18a}{b} - \frac{2b}{a}} \to \frac{3a-b}{18a+6b}$$

3) $$\frac{9 \cdot (0{,}5 \cdot x^2 y^{-2} z)^4}{54 \cdot (4 \cdot x^{-2} y^3 z^{-2})^{-3}} : \frac{36 \cdot (2 \cdot x^2 y^5 z^{-4})^2}{16 \cdot (3 \cdot x^4 y^3 z^{-4})^3} \to \frac{2x^{10}}{z^6}$$

4) $$\frac{\sqrt[2n]{a^{3n+7}}}{\sqrt{\sqrt[n]{a^{5-2n}}}} \cdot \left(\sqrt[4n]{a^2}\right)^{5n-2} \to a^5$$

1) $\frac{2}{1} + \frac{1}{x} - \frac{2}{3} - \frac{1}{4x} - \frac{7}{12} - \frac{3}{4}$

$$\frac{24x + 12 - 8x - 3 - 7x - 9x}{12x} = \frac{9}{12x} = \frac{3}{4x}$$

$\frac{3a}{b}$

2) $$\frac{\frac{(3a)^2 - 6ab + b^2}{3ab}}{\frac{18a^2 - 2b^2}{ab}} = \frac{(3a-b)^2}{3ab} \cdot \frac{ab}{2 \cdot (9a^2 - b^2)}$$

$(3a-b)(3a+b)$

$$\frac{3a-b}{6 \cdot (3a+b)}$$

3) $$\frac{1\,(2^{-1}x^{2}y^{-2}z)^{4}}{6\,(2^{2}x^{-2}y^{3}z^{-2})^{-3}} \cdot \frac{4\,(3x^{4}y^{3}z^{-4})^{3}}{9\,(2x^{2}y^{5}z^{-4})^{2}}$$

$$\frac{2^{-4}\,x^{8}\,y^{-8}\,z^{4}\;4\cdot 3^{3}\,x^{12}\,y^{9}\,z^{-12}}{6\;2^{-6}\,x^{6}\,y^{-9}\,z^{6}\;9\;2^{2}\,x^{4}\,y^{10}\,z^{-8}}$$

$$\frac{2^{2}\,4\cdot 3^{3}\cdot 2^{6}\;x^{8}\,z^{4}\,x^{12}\,y^{9}\,y^{9}\,z^{8}}{2^{4}\cdot 6\;9\cdot 2^{2}\,y^{8}\,z^{12}\,x^{6}\,z^{6}\,x^{4}\,y^{10}}$$

$2\cdot 3$ $\quad 3^{2}$

$$\frac{2^{8}}{2^{7}}$$

$$\frac{2\,x^{10}}{z^{6}}$$

4)

$$\frac{\sqrt[2n]{a^{3n+7}}}{\sqrt[2n]{a^{5-2n}}} \cdot \left(\sqrt[2n]{a}\right)^{5n-2}$$

$$\frac{a^{\frac{3n+7}{2n}}}{a^{\frac{5-2n}{2n}}} \cdot a^{\frac{5n-2}{2n}}$$

$$a^{\frac{3n+7-(5-2n)+5n-2}{2n}}$$

$$a^{\frac{10n}{2n}} \longrightarrow a^{5}$$

$$ld\ \sqrt[3]{0{,}25} = ld\ \sqrt[3]{1/2^2} = ld\ 2^{-2/3}$$

$$-2/3 \cdot \underbrace{ld\ 2} = -2/3$$

$$\log_{2} 2 = x \iff 2^x = 2 \Rightarrow x = 1$$

$$\sqrt{x} - \sqrt{5} = 3$$

$$2 \ln x - \tfrac{1}{2} \ln 16 = 3 \ln 2 + \tfrac{1}{2} \ln x^4 \qquad | \uparrow e^x \quad ↯$$

$$\ln x^2 - \ln 16^{1/2} = \ln 2^3 + \ln (x^4)^{1/2}$$

$$\ln \frac{x^2}{4} = \ln 8 \cdot x^2 \qquad | \uparrow e^x$$

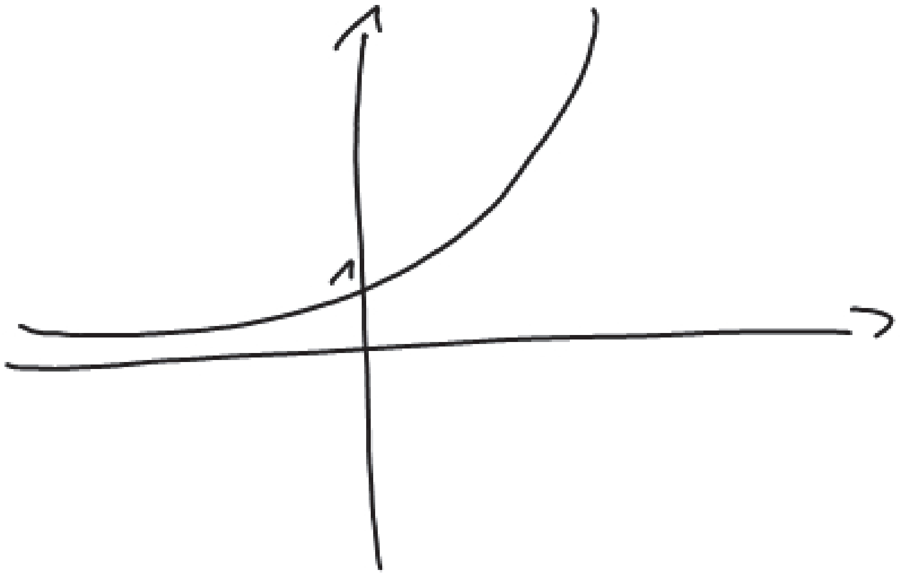
$$\frac{x^2}{4} = \frac{1}{4} x^2 = 8x^2 \qquad | : x^2 \cdot 4$$

$$1 = 32 \Rightarrow \mathbb{L} = \{\}$$

$x = 8$ $\qquad$ $\underbrace{\ln}\, x = \ln 8$

$\log_e 8 = y$

$e^y = 8$



$$f(x) = e^x \quad \rightarrow \quad f'(x) = e^x$$

$$(a^x)' = \ln a \cdot a^x$$

$$g(x) = 42^x = (e^?)^x$$

$$e^? = 42 \qquad ? = \log_e 42 = \ln 42$$

$$\rightarrow g(x) = (e^{\ln 42})^x = e^{\ln 42 \cdot x}$$

$$g'(x) = e^{\ln 42 \cdot x} \cdot (\ln 42 \cdot x)'$$

$$= \ln 42 \cdot 42^x$$

1) $3 \cdot \log(x-y) + \log(x+y) - \frac{1}{2} \cdot \log(x-y)^4$

$\log(x-y)^3 + \log(x+y) - \log((x-y)^4)^{\frac{1}{2}}$

$\log \frac{(x-y)^3 \quad (x+y)}{(x-y)^2} = \log[(x-y) \cdot (x+y)]$

$= \log(x^2 - y^2)$

2) $2 \cdot \ln 2x - 3 \ln 2 + 4 \cdot \ln \sqrt{x} + 2 \cdot \ln\left(\frac{4}{x^2}\right)$

$\ln(2x)^2 - \ln 2^3 + \ln(\sqrt{x})^4 + \ln\left(\frac{4}{x^2}\right)^2$

$\ln 4x^4 - \ln 8 + \ln x^2 + \ln \frac{16}{x^4}$

$\ln \frac{4x^4 \quad x^2 \quad 16}{8 \quad\quad x^4} = \ln(8x^2)$

3) $\log \sqrt[5]{\frac{x^3 y^2}{3\cdot(x+y^2)}} = 1/5 \log \frac{x^3 y^2}{3(x+y^2)}$

$1/5 \cdot \left[\log x^3 + \log y^2 - \log 3 - \log(x+y^2) \right]$

$1/5 \left[3\log x + 2\log y - \log 3 - \log(x+y^2) \right]$

4) $\ln \left[\frac{2\cdot \sqrt{a-2s}}{c^2 \sqrt[4]{d}} \right]^3 = 3\cdot \ln \frac{2\cdot(a-2s)^{1/2}}{c^2\, d^{1/4}}$

$3\cdot \left[\ln 2 + 1/2 \cdot \ln(a-2s) - 2\cdot \ln c - 1/4 \ln d \right]$

$$1)\quad \log \tfrac{1}{100} - \sqrt{e}^{\,\ln 4} + 4^{\operatorname{ld} 3} - 2\cdot \operatorname{ld}(0{,}25)$$

Potenz

$$\log 10^{-2} - \left(e^{1/2}\right)^{\ln 4} + \left(2^{2}\right)^{\operatorname{ld} 3} - 2 \operatorname{ld} 2^{-2}$$

Potenz
Operationsgesetz

$$-2 - e^{1/2\cdot \ln 4} + 2^{2\cdot \operatorname{ld} 3} - 2\cdot(-2)$$

Log

$$-2 - e^{\ln 4^{1/2}} + 2^{\operatorname{ld} 3^{2}} + 4$$

Op. /Gesetz

$$-2 - 4^{1/2} + 3^{2} + 4$$

Potenz

$$\underbrace{-2 - 2 + 9}_{9} + 4$$

2) $10^{2\cdot\log 3} - \ln e^{-2} + \frac{1}{2}\cdot \mathrm{ld}\, 2^4 - e^{\ln 2^3}$

$9 - (-2) + \frac{1}{2}\cdot 4 - 8 = 5$

3) $2^{-3\,\mathrm{ld} 2} - 6\cdot \ln e^{-\frac{1}{3}} + \frac{1}{4}\,\mathrm{ld}\, 2^6 - \frac{1}{2}\log 10^{-3} + e^{\frac{1}{3}\cdot \ln 27}$

$\frac{1}{8} - 6\cdot(-\frac{1}{3}) + \frac{1}{4}\cdot 6 - \frac{1}{2}\cdot(-3) + \sqrt[3]{27}$

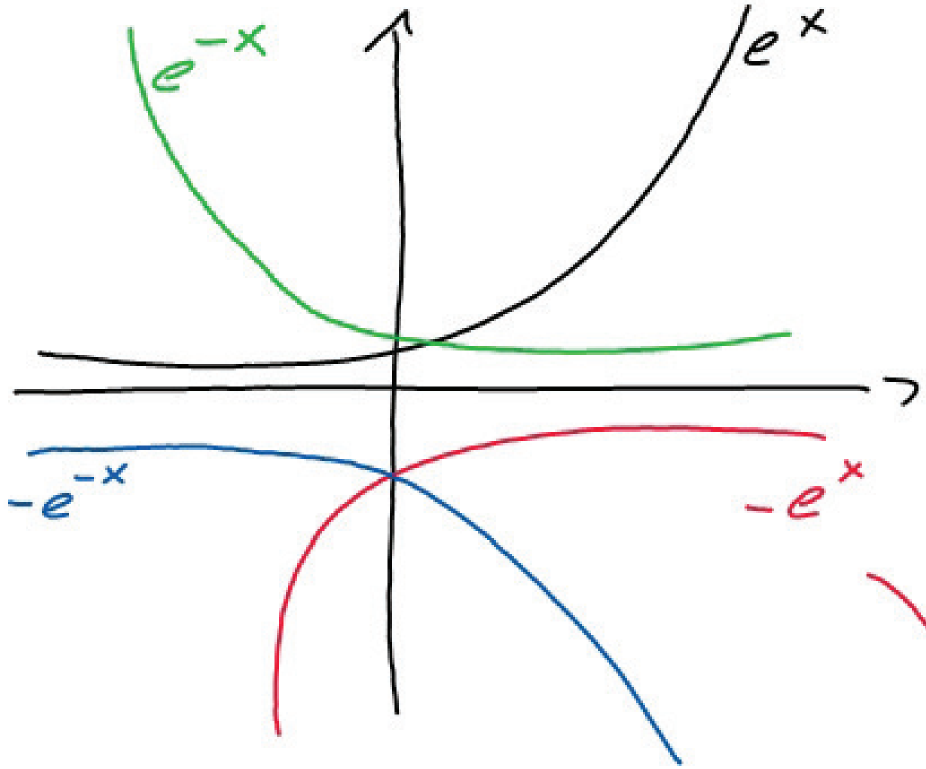
$\frac{1}{8} + 2 + \frac{3}{2} + \frac{3}{2} + 3 = 8\frac{1}{8}$

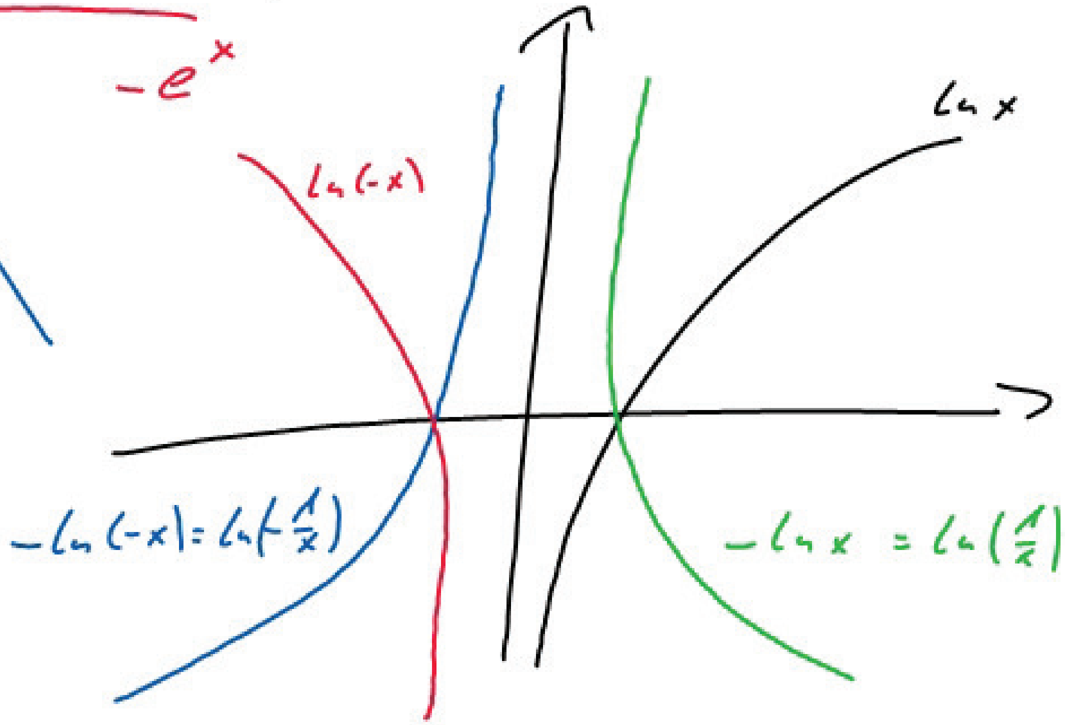
4) $e^{-\frac{1}{2} \ln \frac{1}{9}} + 10^{2 \cdot \log 2^{2}} - 2^{4 \cdot \frac{1}{2} \cdot \mathrm{ld}\, 4} + 2 \log 10^{-3}$

$- 3 \cdot \ln e^{-3} + \frac{1}{4} \mathrm{ld}\, 2^{-8}$

$\left(\frac{1}{9}\right)^{-\frac{1}{2}} = \left(\frac{9}{1}\right)^{\frac{1}{2}} = \sqrt{9} = 3$

$3 + 16 - 16 - 8 + 9 - 2 = 2$


e^{-x}
e^x
$-e^{-x}$
$-e^x$


$\ln x$
$\ln(-x)$
$-\ln(-x) = \ln(-\frac{1}{x})$
$-\ln x = \ln(\frac{1}{x})$

1) $\log x^3 - \log \left(\frac{2}{x}\right)^4 - \log \left(x^{12}\right)^{1/3} = \log (27)^{2/3} + \log (x^4)^{1/4} - \log 6^2$

$$\log \frac{x^3}{\frac{16}{x^4} \quad x^4} = \log \frac{9 \cdot x^2}{6^2} \qquad | \, 10^x$$

$$\frac{x^3 \cdot \cancel{x^4}}{16 \cdot \cancel{x^4}} = \frac{9 \cdot x^2}{6^2} \qquad | : x^2 \cdot 16$$

$$\frac{x^3}{x^2} = \frac{\cancel{9} \cdot 16}{\cancel{36}_{4}}$$

$$x = 4 \qquad \Rightarrow \quad \mathbb{L} = \{4\}$$

2) $\ln 4^3 - \ln\left(\frac{16}{x^4}\right)^{1/2} + \ln 8^2 = \ln(x^4)^{3/2} - \ln\left(\sqrt[4]{\frac{1}{x}}\right)^8 - \ln\left(\frac{1}{4}\right)^2$

$$\ln \frac{4^3 \cdot 8^2}{\sqrt{\frac{16}{x^4}}} = \ln \frac{x^6}{\left(\left(\frac{1}{x}\right)^{1/4}\right)^8 \left(\frac{1}{4}\right)^2} \quad \Big| \uparrow e^x$$

$$\frac{2^6 \cdot 2^6}{\frac{2^2}{x^2}} = \frac{x^6}{\frac{1}{x^2} \cdot \frac{1}{2^4}}$$

$$\frac{2^{12} \cdot \underline{x^2}}{2^2} = x^6 \cdot x^2 \cdot \underline{2^4} \quad \Big| :2^4 : x^2$$

$$\frac{2^{12}}{2^2 \cdot 2^4} = 2^6 = x^6 \quad \Rightarrow x = 2$$

$\mathbb{L} = \{2\}$

$$x - 3 = 2\cdot\sqrt{2-3x} - 4 \qquad | +4$$

$$x + 1 = 2\cdot\sqrt{2-3x} \qquad | \uparrow^2$$

$$(x+1)^2 = 4\cdot\left(\sqrt{2-3x}\right)^2$$

$$x^2 + 2x + 1 = 8 - 12x \qquad | -8 \; +12x$$

$$x^2 + 14x - 7 = 0$$

$$x_{1,2} = -7 \pm \sqrt{7^2+7} = -7 \pm \sqrt{56}$$

$$\frac{2x-5}{3-x} = \frac{x+4}{x-1} \qquad \big|\cdot(3-x)\cdot(x-1)$$

$$(2x-5)(x-1) = (x+4)(3-x)$$

$$2x^2-2x-5x+5 = -x^2-4x+3x+12$$

$$2x^2-7x+5 = -x^2-x+12 \qquad \big| +x^2+x-12$$

$$3x^2-6x-7=0 \qquad \big| :3$$

$$x^2-2x-\tfrac{7}{3}=0$$

$$x_{1/2} = 1 \pm \sqrt{1^2+\tfrac{7}{3}} = 1 \pm \sqrt{\tfrac{10}{3}}$$

1) Vieta $\quad 2x^2 - 8x - 10 = 0 \quad | : 2$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0 \quad \Rightarrow \quad \mathbb{L} = \{-1; 5\}$$

2) p-q : $\quad 3x^2 - 9x + 30 = 0 \quad | : 3$

$$x^2 - 3x + 10 = 0$$

$$x_{1/2} = 3/2 \pm \sqrt{\underbrace{9/4 - 10}_{<0}} \quad \Rightarrow \quad \mathbb{L} = \{\}$$

3) Q.E. : $\quad x^2 + 12x + 32 = 0$

$$(x+6)^2 - 36 + 32 = 0 \quad | +4$$

$$(x+6)^2 = 4 \quad | \sqrt{}$$

$$x+6 = \pm 2$$

$$x_1 = -8 \quad \vee \quad x_2 = -4$$

4) $f(x) = -2(x^2-6x+9) = -2\cdot(x-3)^2$

$S_y(0|-18)$; $S_x(3|0)$

↳ Scheitelpunkt HP?

-> streck. (entl. y-Achse) und nach unten geöffnet

-> symmetrisch zu $x=3$

5) $g(x) = \frac{1}{2}(x^2+20x+64) = \frac{1}{2}\cdot\underbrace{(x+16)(x+4)}_{6\cdot(-6)}$

$S_y(0|32)$, $S_{x_1}(-16|0)$; $S_{x_2}(-4|0)$

$S(-10\,|\,f(-10)) = S(-10|-18)$ ↑ flacher

↳ TP?

7) $x^4 - 24x^2 - 25 = \underbrace{(x^2 - 25)}_{\mathbb{L} = \{\pm 5\}} \underbrace{(x^2 + 1)}_{\neq 0}$

8) $x^8 - 17x^4 + 16 = \underbrace{(x^4 - 16)}_{\pm 2} \underbrace{(x^4 - 1)}_{\pm 1}$ $\Big\}$ $\mathbb{L} = \{\pm 1, \pm 2\}$

9) $x = 2 \cdot \sqrt{6 - x} + 6 \qquad | -6$

$x - 6 = 2 \cdot \sqrt{6 - x} \qquad | \uparrow^2$

$x^2 - 12x + 36 = 4 \cdot (6 - x) = 24 - 4x \qquad | -24 + 4x$

$x^2 - 8x + 12 = (x - 6)(x - 2)$

$x_1 = 6 \quad \vee \quad x_2 = 2$

10) $\dfrac{x^2+4}{29+x^2} = \dfrac{x^2}{4+2x^2} \qquad | \cdot (29+x^2)(4+2x^2)$

$(x^2+4)(2x^2+4) = x^2 \cdot (x^2+29)$

$2x^4+8x^2+4x^2+16 = x^4+29x^2 \qquad | -x^4-29x^2$

$\cancel{x^4 - 17x^2 - 16 = 0} \qquad x^4 - 17x^2 + 16 = 0$

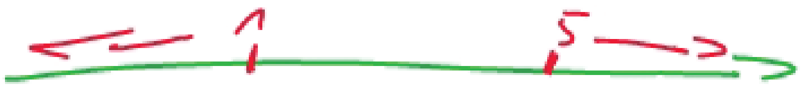
$(x^2-16)(x^2-1) = 0$

$L = \{\pm 1; \pm 4\}$

$|4x-12|>8$

$x>3$: $4x-12>8$ δ^+	$x\leq 3$: $-(4x-12)>8$ δ^-	F
$4x-12>8 \quad \vert +12$ $4x>20 \quad \vert :4$ $x>5$	$-4x+12>8 \quad \vert -12$ $-4x>-4 \quad \vert :(-4)$ $x<1$	R
$x>5$	$x<1$	E
$x=6$ $\quad \vert 24-12\vert=12>8$ ✓	$x=0$ $\quad \vert 0-12\vert=\vert 12\vert=12>8$ ✓	P

$\mathbb{L}=\{x\in\mathbb{R} \mid x<1 \vee x>5\}$ L



3) $\dfrac{3x+2x^2}{6-2x} > 1 - x \qquad | \cdot (6-2x)$

$x > 3$ $\qquad \delta^-$	$x < 3$ $\qquad \delta^+$
$3x + 2x^2 < (1-x)\cdot(6-2x)$ $3x + 2x^2 < 6 + 2x^2 - 6x - 2x$ $11x < 6$ $x < 6/11$	... $x > 6/11$
$x < 6/11 \lor x > 3$	$x > 6/11 \land x < 3$
$x = 0 \quad 0/6 > 1 \quad f$	$x = 1 \quad \frac{5}{4} > 0 \quad \checkmark$

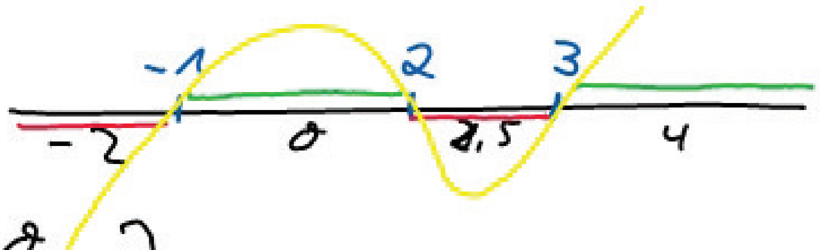
$\mathbb{L} = \{ x \in \mathbb{R} \mid x > 6/11 \land x < 3 \}$

5) $x^3 - 4x^2 + x + 6 > 0$ $\qquad x = -1$

$$(x^3 - 4x^2 + x + 6) : (x+1) = \underbrace{x^2 - 5x + 6}_{(x-2)(x-3)}$$

$$\begin{array}{l}
-(x^3 + x^2) \\
\hline
\quad / \quad -5x^2 + x + 6 \\
\quad -(-5x^2 - 5x) \\
\hline
\qquad / \quad 6x + 6 \\
\qquad -(6x + 6) \\
\hline
\qquad\quad / \quad /
\end{array}$$

$(x+1)(x-2)(x-3) > 0$



$x = -2$: $(-)\ (-)\ (-)\ < 0$

$x = 0$: $(+)\ (-)\ (-)\ > 0$

$x = 2{,}5$: $(+)\ (+)\ (-)\ < 0$

$x = 4$: $(+)\ (+)\ (+)\ > 0$

$\mathbb{L} = \{x \in \mathbb{R} \mid x > -1 \wedge x < 2 \vee x > 3\}$